



Divergence in Labour Force Growth in Open Economies: *Should Wages and Prices Grow Faster in Germany?*

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Abstract

We develop a model which shows that wages, prices, and the real per-capita income should grow more rapidly in open economies with low labour force growth. Otherwise, their trade partners experience rising unemployment and/or trade deficits. We apply this framework to Germany, which has exhibited modest labour force growth, except at the moment of reunification. Goods being differentiated by country of origin (Armington's hypothesis), low labour force growth limits German production and should lead to rising prices and wages relative to other countries. This mechanism is magnified by the low price elasticity of demand for German goods. Hence, German wage moderation could constrain other countries' policy options. Simulations using an extended version of the model suggest that (i) disparities in labour force growth have had a significant impact on unemployment within the Eurozone, potentially contributing to the severe economic crisis faced by Southern European countries between 2010 and 2015, and (ii) the demographic shock following reunification could explain a large part of the German economic challenges from 1995 to 2005.

Keywords Germany · Inflation · Labour force growth · Trade · Wages

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Introduction

In the past fifty years, the German economy has exhibited several distinctive characteristics. First, inflation in Germany has been significantly lower than that in other advanced economies. As regards Eurozone countries, the resulting gain in price competitiveness has been only partially offset by the appreciation of the German currency (the Deutsche Mark) until 1999, and the German effective real exchange rate has depreciated over the long term. Second, German unemployment has permanently been lower than that of most other advanced economies, except during the 15 years following reunification in 1990. Third, Germany has consistently maintained a trade surplus, although this surplus diminished in the years immediately following reunification. Finally, Germany has experienced a significantly smaller increase in its labour force compared to other advanced economies, except for a one-off shock in the year of reunification. Several explanations have been proposed for the first three characteristics of the German economy: a stable and strict monetary policy, the social consensus guiding the employer–employee relations, a quality and productivity gap with other countries, and a high propensity to save among German households.

Based on these observed facts, the model developed here has a double objective. First, it provides a new theoretical channel through which differences in labour force growth across countries have an impact on the changes in nominal wages, prices and real income in those countries. Second, it offers an additional explanation for the German economic performance in comparison with other advanced economies

Using a simple theoretical model, we first demonstrate (i) that, assuming perfect competition in labour markets (full employment) and balanced trade in all countries, a country with a relatively low increase in its labour force should experience higher growth in wages, prices, and real income compared to its trade partners, and (ii) that if these outcomes are prevented, the low increase in the country's labour force generates growing unemployment and/or trade deficits for its trade partners. These results are subsequently utilised to explain the economic differences and relationships between Germany and other advanced countries.

The intuition behind these results is as follows. When goods are differentiated by their country of origin (Armington's hypothesis), a low increase in a country's labour force limits its production, leading to an increase in the price of its goods relative to those of other countries. This effect is amplified if the price elasticity of demand for its goods is low, as a low elasticity necessitates a substantial rise in prices to curb demand. Ultimately, in the model's general equilibrium, higher prices coincide with higher wages compared to other countries. Moreover, as the consumer price index includes imported goods whose prices rise less than domestic prices and wages, the country's real income per capita increases relative to other countries.

Now, if market imperfections inhibit the relative decline in prices and wages in the country's trade partners, this diminishes the demand for their goods, generating rising unemployment in those countries. Additionally, if a share of the country's



total income moves to the trade partners, this results in a trade surplus for the country and can potentially reduce unemployment in the other countries. This is what has been observed in the economic relationships between Germany and other EU countries before German reunification and after 2005, the effects being intensified by the low price elasticity of demand for German goods.

The major analytical results of the paper are in a first step derived from a simple stylised model featuring two advanced economies, each producing country-specific goods, with one of them characterised by low labour force growth. This simple framework is convenient for demonstrating the results *ceteris paribus*, but it cannot account for certain developments that may alter the outcomes, such as the extensive offshoring of low-skilled production stages to emerging countries—a trend that has characterised German industries since the mid-1990s—and the differences between Eurozone and non-Eurozone countries in their relations with Germany.

To analyse the mechanisms within a more realistic framework, we construct, in a second step, an extended model in which (i) there are three advanced regions (Germany, the Eurozone, and non-Eurozone advanced economies) and one emerging region, and (ii) we introduce increasing offshoring of low-skilled production stages to emerging countries. This model is subsequently utilised to simulate several scenarios: (1) a situation characterised by differences in labour force growth, competitive labour markets, and balanced trade; (2) the combination of differences in labour force growth with offshoring, wage rigidity in unskilled labour markets, and income transfers; and (3) a counterfactual scenario assuming no difference in labour force growth across countries within the second scenario. The simulations suggest that (i) the impact of differences in labour force growth on unemployment in Eurozone countries has been significant and may have played an important role in the severe economic crisis experienced by Southern European countries from 2010 to 2015, and (ii) the demographic shock in Germany following reunification could explain a large part of the economic turmoil in Germany from 1995 to 2005.

The parameters and exogenous variables for the simulations have been calculated in accordance with the structure of the model and the groups of countries considered (Germany, the Eurozone, and non-Eurozone advanced economies). Data from the World Input-Output Database (WIOD) have been used to calculate time series for the offshoring intensities in the three regions, taking into account the input-output linkages between countries along global value chains. The same database has been utilised to estimate export equations, thereby deriving the price elasticity of exports and the elasticity of substitution between tradable goods produced in the three regions.

“[Stylised Facts and Literature](#)” section presents the major stylised facts and briefly reviews the literature on the subject. “[A stylised Model](#)” section develops a simple general equilibrium model to determine the key analytical results. “[The Extended Model](#)” section introduces the extended model used for the simulations of the German case. “[Simulations](#)” section describes the strategies and data employed in the simulations and reports their results. “[Discussion and Conclusion](#)” section summarises the major findings and presents some conclusions.



Stylised Facts and Literature

Stylised Facts

We distinguish between three regions: Germany, other advanced euro area economies, called the *Eurozone*, and advanced non-Eurozone countries, collectively referred to as the *North*. This distinction is motivated by the different developments in Eurozone and North, as well as by the fact that Germany and Eurozone have shared the same currency since 1999. The term “*Eurozone*” will be used even when referring to the period before the introduction of the euro.

Low Growth of the German Labour Force

Structurally, Germany has exhibited a lower increase in its labour force compared to other advanced economies (Fig. 1a). This can be observed before (1970–1989) and after (since 1990) reunification. However, reunification resulted in a one-time increase in the German labour force, which rose from 30.7 million in 1990 to 39.6 million in 1991.

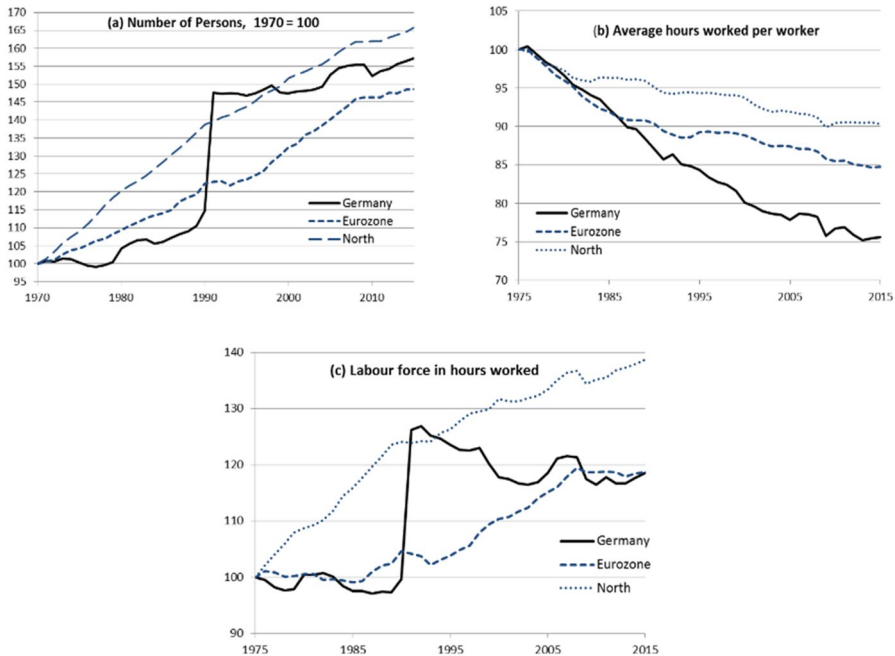


Fig. 1 Growth in the labour force, 1970–2015 (initial year = 100). *Notes:* 1970–1989: German Federal Republic. Since 1990: Germany. *Eurozone* = Austria, Belgium, France, Finland, Greece, Ireland, Italy, Netherlands, Portugal, Spain. *North* = Australia, Denmark, Canada, Japan, New Zealand, Norway, Sweden, UK, US. *Data source* OECD (*Annual Labour Force Statistics*)



When comparing labour force developments, it is also important to consider the reduction in average annual hours worked, which has been more pronounced in Germany than in other advanced countries (Fig. 1b). This suggests that the labour force growth gap between Germany and other advanced economies has even been more pronounced in terms of hours worked than in terms of the number of persons (Fig. 1c).

Low Inflation and the Real Exchange Rate

Since 1970, inflation has consistently been lower in Germany than in other advanced countries, except (i) during the three years following reunification, and (ii) since 2008, due to the decrease in inflation in Southern Europe. From 1970 up to 2015, prices, as measured by the GDP deflator, have quintupled in the Eurozone compared to Germany and doubled in the North (Fig. 2). The appreciation of the German currency (D-Mark until 1999 and euro thereafter, Fig. 3) has counteracted the inflation gap, though the total impact varies across these regions (Fig. 4). In the long term, German prices have decreased compared to Eurozone prices, particularly between 1995 and 2005, when German prices fell by almost 25% relative to Eurozone prices. In contrast, over the long term, the real exchange rate of Germany vis-à-vis

Fig. 2 Relative price (Area/Germany). *Notes* The price index is the GDP deflator. The German exchange rate with each area is the weighted average of the exchange rates indices with the area's countries, the weights being the yearly country shares in the area's GDP

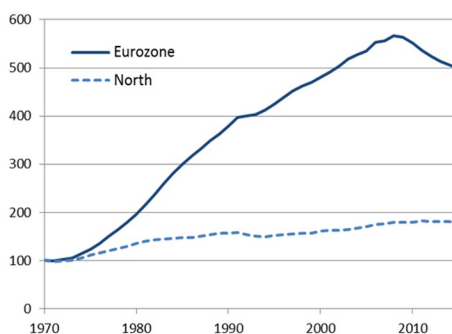
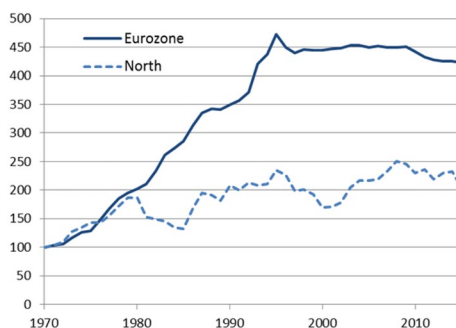


Fig. 3 German nominal effective exchange rate. (Germany = 100; initial year = 100). *Notes* The variation of the exchange rate between Germany and the Eurozone since 1999 reflects the changes in countries' weights



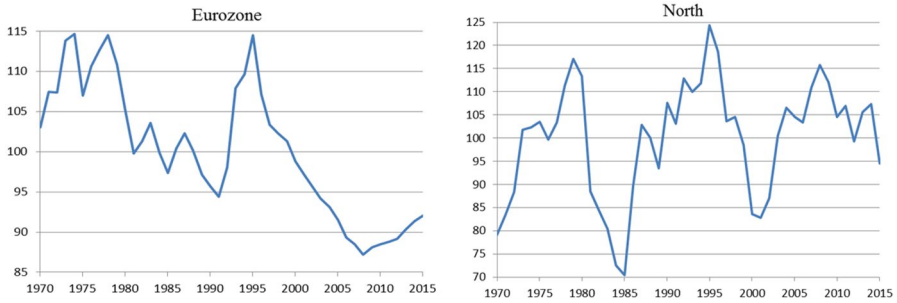


Fig. 4 Germany's real effective exchange rate (REER) (1970–2015; 100 = average level). *Notes* A decline denotes a depreciation of the German REER. Data source for Figs. 2, 3, and 4: IMF (*World Development Indicators*)

non-Eurozone advanced economies has slightly increased, albeit with large fluctuations due to the volatility of the US dollar (Fig. 4).

Low Unemployment Except in the Post-reunification Period

As shown in Fig. 5, Germany has experienced lower unemployment over the last fifty years compared to Eurozone countries, except during the period from 2001 to 2008. Compared to other advanced economies, the evidence is mixed: German unemployment was lower until the early 1980s, similar from the early 1980s to the early 1990s, higher from the early 1990s to 2008 and lower since then. A notable trend following reunification in 1990 is that German unemployment increased relative to both the Eurozone and the North until 2006 and then decreased thereafter. Finally, it is important to note that in all three areas, unemployment primarily affects less skilled workers.

Permanent Trade Surplus and Large Offshoring

In the last fifty years, Germany has consistently maintained a substantial trade and current account surplus with both non-Eurozone and Eurozone advanced economies, except during the eight years following reunification with the latter area

Fig. 5 Unemployment gap relative to Germany. *Notes* The unemployment gap is the difference in unemployment rates between the respective area and Germany. The unemployment rate is the ratio of unemployment to the labour force in each area. Data source OECD Stat. (*Population and Labour Force*)

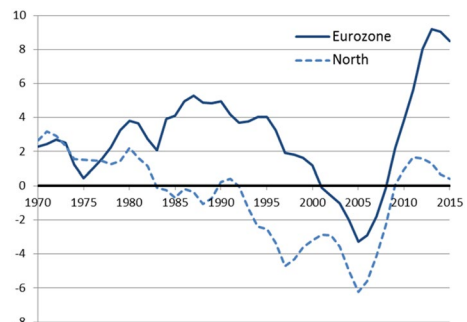


Fig. 6 German current account surplus (in % of German GDP). Data sources: Deutsche Bundesbank

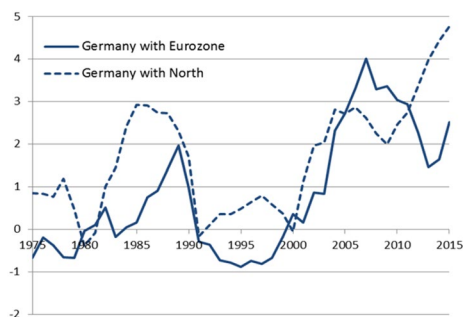
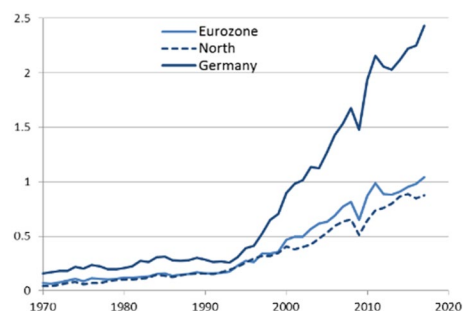


Fig. 7 Imports of intermediate goods from emerging countries (in % of area GDP). Data sources: The CHELEM database



(Fig. 6). Since the 2008 financial crisis, the surplus with non-Eurozone countries has continued to grow, whereas the surplus with the Eurozone has decreased due to the severe recession in Southern Europe. Additionally, since the mid-1990s, Germany has experienced a significant increase in offshoring to emerging countries which is much larger than that of other advanced economies (Fig. 7).

Related Literature: Demographic Changes and German Economic Performance

The impact of demographic changes on relative prices across countries has primarily been analysed through the influence of ageing on the real effective exchange rate (REER), as surveyed by Hassan et al. (2011). The key mechanism can be summarised as follows: According to the life cycle hypothesis (LCH), individuals save during their working years and dissave once retired. When applying the LCH to open economies with mobile capital flows and differing age structures, countries with a higher proportion of retired households tend to save less and therefore experience net capital inflows, whereas countries with a larger proportion of working households experience net capital outflows. As capital inflows lead to exchange rate appreciation, the REER tends to appreciate in ageing countries.¹ Georges et al. (2013) incorporate these life-cycle elements with an Armington trade structure in

¹ This impact can be offset by the fact that countries with ageing populations tend to invest less. Then, a decrease in domestic investment may lead to capital outflows and REER depreciation (Fougère & Mérette 1998).

a multi-country overlapping generations (OLG) model and demonstrate that the rapid ageing of Northern countries results in an appreciation of their REER relative to Southern countries. Another channel through which ageing affects the REER is the increased demand for non-tradable services by the elderly, which raises the country's price index via the Balassa–Samuelson effect, thereby impacting its REER (Van Ewijk & Volkerink 2012; Groneck & Kaufmann 2017). For net creditor countries, a real appreciation may also be induced by a declining birth rate and the resulting increase in wealth per capita, which influences consumption (Aloy & Gente 2009). The impact of ageing on the REER seems to be confirmed by empirical studies, including those of Anderson & Osterholm (2006) and Salim & Hassan (2012).

Similar to our study, Fedotenkov et al. (2014) examine international spillover effects stemming from a decline in the labour force in one country. However, their focus is on the direction of capital flows between countries, whereas ours is on the consequences of labour force divergence on relative prices, skill premia and unemployment. Their analysis employs a two-country OLG model with country-specific intermediate goods. In this model, capital flows are influenced by two opposing effects: If the labour force decreases in the Home country, capital *ceteris paribus* flows to the Foreign country, where it is more productive. However, due to the decline in the domestic labour force, the Home country's intermediate good becomes scarcer, leading to an appreciation of its real exchange rate. This appreciation *ceteris paribus* leads to a capital inflow to the Home country. Which effect dominates depends on the substitutability between the countries' intermediate goods. Both our study and that of Fedotenkov et al. (2014) endorse the view that the consequences of international divergence in labour force growth should be analysed in a model where traded goods are imperfect substitutes, following Armington's hypothesis. Beyond this, we address distinct research questions. To our knowledge, the demographic channel proposed in our paper—namely, the impact of labour force growth on countries' prices, skill premia, and potentially unemployment—has not been explored previously in the literature.²

In addition to the theoretical results related to labour force divergence, our study applies these findings to the German context, contributing to the literature on key aspects of German economic performance. From the early seventies up to the COVID pandemic, Germany's economic performance relative to other advanced economies can be broadly classified into three phases (Giersch et al. 1992, chap. 5, for the 1970s and 1980s; Carlin & Soskice 1997, for the 1980s and 1990s). From the early 1970s until reunification, Germany was characterised by low inflation, sustained revaluation of the D-Mark, and persistent trade surpluses. Additionally, Germany was less affected by the two oil shocks compared to most other advanced

² Docquier et al. (2019) and Mennuni (2019) analyse the economic impact of changes in the labour force, but they focus on the influence of compositional changes (in age, education, gender and migration) on the wages of the different skill and age groups for the former, and on growth and business cycle for the latter. In a Heckscher–Ohlin model with capital and labour, Choi (2008) shows that labour growth raises the price of the capital intensive good and improves the North's terms of trade. He (2018) looks at the (reverse) impact of inflation on population growth by constructing a model in which monetary policy affects fertility choices via changes in the nominal interest rate.



countries. In the decade following reunification, Germany was deemed the ‘sick man’ of Europe, marked by rising unemployment and low growth. From 2005 up to the COVID pandemic, Germany has witnessed a new ‘economic miracle’, with consistently decreasing unemployment, economic growth rates surpassing those of other European countries, and large trade surpluses.

The determinants of German economic performance, as highlighted in the literature, have varied over time. The characteristics of Germany in the 1970s and 1980s—low inflation, Deutsche Mark appreciation, trade surplus, and relatively low unemployment—have been attributed to three main factors: (i) a monetary policy focused on long-term stability, (ii) bargaining practices based on social consensus, and (iii) sustained increases in productivity and quality. The economic difficulties following reunification were linked to a loss of competitiveness due to high wages and low labour market flexibility during a period of high unification-related public expenditure. The strong performance of the German economy since 2005 has attracted considerable attention in the literature, with several explanations proposed. The first suggests that labour market reforms (Hartz reforms implemented from 2003 to 2005) marked the beginning of enhanced German competitiveness (e.g. Kirkegaard 2014; Rinne & Zimmermann 2013). Dustmann et al. (2014), however, noted that improvements in competitiveness and trade began in the 1990s and may result from significant changes in labour market structure prior to the Hartz reforms (see also Carlin & Soskice 2009). Burda & Hunt (2011) show that the decision of German firms to reduce working hours rather than the workforce was key to explaining low unemployment following the 2008 financial crisis. Gehrke et al. (2019) emphasise the importance of ‘performance shocks’ related to labour market reforms. Beissinger et al. (2016) suggest that the improvement in German competitiveness and trade may primarily result from the offshoring activities of firms, with labour market reforms mitigating offshoring-related unemployment. Bachmann et al. (2014) find that outward FDI to Central and Eastern Europe significantly reduced employment security in Germany. Bradley & Kügler (2019) report that the Hartz reforms shortened unemployment duration and exerted downward pressure on wages, but did not affect unemployment.³

Changes in international price competitiveness are typically mirrored by changes in the real effective exchange rate (REER). Those changes may account for the imbalances within the Eurozone and Germany’s distinctive position, as price divergences can no longer be mitigated through exchange rate adjustments, as argued by Belke & Dreger (2013). Related to this is the ongoing policy debate on whether Germany should foster higher growth in wages and prices, which could reduce other countries’ relative prices and facilitate adjustments for countries with imbalances (Economist 2017; Sinn 2014). Another explanation for the shifts in REERs within the Eurozone is the economic catch-up in Southern Europe combined with the Balassa–Samuelson effect (Diaz del Hoyo et al. 2017).

³ See also Bauer & King (2018), Carrillo-Tudela et al. (2021) and Hochmuth et al. (2021) for analyses of the impacts of the Hartz reforms on the German labour market.



In these debates, to our knowledge, the fact that Germany has experienced a persistently low increase in its labour force relative to other countries has not been considered.

A stylised Model

We construct a stylised model that enables us to determine our major findings *ceteris paribus*. This model seeks to illustrate the mechanisms outlined in the introduction. Under conditions of full employment, a country with high labour force growth will experience an increase in production compared to a country with lower labour force growth. If goods are country-specific and thus not perfectly substitutable, the former country must experience a relative decrease in prices compared to the latter in order to sell its products.

We assume that the countries' labour forces are determined independently from the variations in prices, wages, production and per capita income. Consequently, the most straightforward way to address our research question is to use a comparative-static general equilibrium framework and to introduce labour force growth exogenously.

There are two countries, Home and Foreign, with Foreign values denoted by an asterisk (*). Each country consists of two sectors. Following Armington's hypothesis, the tradable sector (*T*) produces tradable goods differentiated by their country of origin. The non-tradable sector (*NT*) provides a single non-tradable service.

The Home and Foreign countries, respectively, produce *N* and *N** tradable goods using both skilled and unskilled labour. Production of the non-tradable service employs only unskilled labour. The markets for goods and services are competitive.

Cross-Country Differences in Labour Force Growth

The endowments with unskilled and skilled labour at time *t* are denoted $\bar{L}(t)$ and $\bar{H}(t)$ for the Home country and $\bar{L}^*(t)$ and $\bar{H}^*(t)$ for the Foreign country. To focus on the impact of divergence in labour force growth, we assume that the relative endowments $\bar{\lambda}$ and $\bar{\lambda}^*$ remain unchanged over time in each country:

$$\frac{\bar{H}(t)}{\bar{L}(t)} = \bar{\lambda}, \quad \frac{\bar{H}^*(t)}{\bar{L}^*(t)} = \bar{\lambda}^* \quad (1)$$

Consequently, in each country, both skilled and unskilled labour grow at the same exogenous rates, denoted as *n* for the Home country and *n** for the Foreign country:

$$\bar{L}(t) = \bar{L}(0) \times e^{nt}; \bar{H}(t) = \bar{H}(0) \times e^{nt} \quad (2)$$

$$\bar{L}^*(t) = \bar{L}^*(0) \times e^{n^*t}; \bar{H}^*(t) = \bar{H}^*(0) \times e^{n^*t}$$

The growth rate of the labour force is lower in Home than in the Foreign country:



$$n < n^*$$

(3)

Demand for Goods and Services

The households' utility functions in each country are given by:⁴

$$u = y_{NT}^{1-\beta} y_T^\beta; u^* = y_{NT^*}^{1-\beta} y_T^{*\beta}, \quad 0 < \beta < 1 \quad (4)$$

with $y_T = \left(\sum_{i=1}^N a y_i^\theta + \sum_{i^*=1}^{N^*} a^* y_{i^*}^\theta \right)^{1/\theta}$ and $y_T^* = \left(\sum_{i=1}^N a y_i^{*\theta} + \sum_{i^*=1}^{N^*} a^* y_{i^*}^{*\theta} \right)^{1/\theta}$.

y_{NT} and y_{NT^*} denote the representative Home and Foreign households' consumption of the non-tradable services, and y_T and y_T^* the quantity indices comprising the Home and Foreign households' consumptions of country-specific tradable goods y_i and y_{i^*} , $i = 1 \dots N$, $i^* = 1 \dots N^*$. The parameters a and a^* depict the respective attractiveness of the Home and Foreign tradable goods, assumed identical inside each country for the sake of simplicity. Finally, $\sigma = (1 - \theta)^{-1}$ is the elasticity of substitution between tradable goods.

The utility function defines the consumer price P and P^* in each country:

$$P = (p_{NT})^{1-\beta} (P_T)^\beta; P^* = (p_{NT^*})^{1-\beta} (P_T^*)^\beta \quad (5)$$

with $P_T = \left(a^\sigma \sum_{i=1}^N p_i^{1-\sigma} + a^{*\sigma} \sum_{i^*=1}^{N^*} p_{i^*}^{1-\sigma} \right)^{1/(1-\sigma)}$

where p_{NT} and p_{NT^*} , respectively, denote the price of the non-tradable service in the Home and Foreign country, P_T is the price index of tradable goods, and p_i is the price of tradable good i .

Production

In both countries, the non-tradable service NT is produced with unskilled labour. One unit of unskilled labour produces one unit of NT . Consequently:

$$Y_{NT} = L_{NT}, \text{ and } Y_{NT}^* = L_{NT}^*$$

where Y_{NT} and Y_{NT}^* are the quantity of non-tradable services produced in the Home and the Foreign country, and L_{NT} and L_{NT}^* the unskilled labour utilised in their production.

The sector of tradables (T) comprises country-specific goods, $i = 1 \dots N$ and $i^* = 1 \dots N^*$, produced with both skilled and unskilled labour. Goods i and i^* in the tradable sector are, respectively, produced in the Home and Foreign country with the Cobb–Douglas technologies:

⁴ To simplify the notation, the time index is omitted wherever possible.



$$Y_i = A(L_i)^\alpha (H_i)^{1-\alpha}; Y_{i*} = A^* (L_{i*}^*)^\alpha (H_{i*}^*)^{1-\alpha} \quad (6)$$

where L_i and L_{i*}^* (resp. H_i and H_{i*}^*) are the unskilled (resp. skilled) labour utilised in the Home and Foreign country in the production of goods i and i^* .

In the sector of tradables, the total factor productivity (TFP) is the same for all goods produced inside one country, but differs between countries: A in Home and A^* in Foreign. In contrast, the coefficient α defining the factor intensity is the same for all tradable goods whatever their country, which ensures no Heckscher–Ohlinian specialisation in trade.

Equilibrium in Perfect Competition

Assume that labour markets are perfectly competitive. Consequently, the unit wages of skilled and unskilled labour, denoted as w_H and w_L in the Home country and w_H^* and w_L^* in the Foreign country, ensure full employment of both types of labour in each country.

Given that the technology is identical for all tradable goods produced within the same country, the prices of tradables goods are uniform in the Home country and in the Foreign country. Therefore, the production price of tradable goods at time t is $p_T(t)$ in the Home country and $p_T^*(t)$ in the Foreign country.

The full employment equilibrium of the model is characterised by balanced trade between the two countries and by the following relations (Appendix A):

$$\frac{p_T(t)}{p_T^*(t)} = \frac{a}{a^*} \left(\frac{N}{N^*} \right)^{\frac{1}{\sigma}} \left(\frac{A^*}{A} \right)^{\frac{1}{\sigma}} \left(\frac{\bar{\lambda}^*}{\bar{\lambda}} \right)^{\frac{1-\alpha}{\sigma}} \left(\frac{\bar{L}^*(0)}{\bar{L}(0)} \right)^{\frac{1}{\sigma}} \times e^{\bar{\gamma}t} \quad (7)$$

$$\frac{w_L(t)}{w_L^*(t)} = \frac{a}{a^*} \left(\frac{N}{N^*} \right)^{\frac{1}{\sigma}} \left(\frac{A^*}{A} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\bar{\lambda}^*}{\bar{\lambda}} \right)^{\frac{(1-\alpha)(\sigma-1)}{\sigma}} \left(\frac{\bar{L}^*(0)}{\bar{L}(0)} \right)^{\frac{1}{\sigma}} \times e^{\bar{\gamma}t} \quad (8)$$

$$\frac{w_H(t)}{w_H^*(t)} = \frac{a}{a^*} \left(\frac{N}{N^*} \right)^{\frac{1}{\sigma}} \left(\frac{A^*}{A} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\bar{\lambda}^*}{\bar{\lambda}} \right)^{\frac{1+\alpha(\sigma-1)}{\sigma}} \left(\frac{\bar{L}^*(0)}{\bar{L}(0)} \right)^{\frac{1}{\sigma}} \times e^{\bar{\gamma}t} \quad (9)$$

with:

$$\bar{\gamma} = \frac{n^* - n}{\sigma} > 0 \quad (10)$$

Proposition 1. Assume a given relative attractiveness a/a^* , a given relative number of goods N/N^* , a given relative total factor productivity A/A^* , and given relative factor endowments $\bar{\lambda}$ and $\bar{\lambda}^*$. If the growth rate of the Home country's labour force n is lower than that of the Foreign country n^* , then at the full employment equilibrium:



1) The tradables' relative price p_T/p_T^* and the relative wages w_L/w_L^* and w_H/w_H^* increase with time at rate $\bar{\gamma} > 0$, the relative consumer price P/P^* at rate $(1 - \beta)\bar{\gamma}$, and the relative real income per head i/i^* as well as the relative real unskilled wage v_L/v_L^* and the relative real skilled wage v_H/v_H^* at rate $\beta\bar{\gamma}$.

2) Those rates increase with the difference in labour force growth $n - n^*$.

3) The smaller the elasticity of substitution σ , the higher those rates.

Proof See Relations (7) – (10). Refer to Appendix A for detailed derivations concerning consumer prices, relative real wages, and relative real income per capita.

Proposition 1 demonstrates that, at full employment in both countries, the country with the lower labour force growth rate (Home) experiences a sustained increase in its prices, as well as in its real and nominal wages and its real income per capita, relative to the other country. These results can easily be extended to encompass multiple countries, where the hierarchy of price and wage growth is the inverse of the hierarchy of labour force growth.

Proposition 2. For given relative factor endowments $(\bar{\lambda}, \bar{\lambda}^*)$, the tradables' relative price p_T/p_T^* is

1) an increasing function of the Home relative attractiveness a/a^* and of the Home relative number of goods N/N^* .

2) a decreasing function of the Home relative TFP A/A^* .

Proof See Relation (7).

Proposition 3. For given relative factor endowments $(\bar{\lambda}, \bar{\lambda}^*)$, the relative wages w_L/w_L^* and w_H/w_H^* are

1) increasing functions of the Home relative attractiveness a/a^* and of the Home relative number of tradables N/N^* .

2) increasing functions of the Home relative TFP A/A^* , if the elasticity of substitution σ is higher than one, and a decreasing function if σ is less than one.

Proof See Relations (8) and (9).

Propositions 2 and 3 indicate that the rise in the Home country's relative prices and wages can be counterbalanced by an increase in the Foreign country's relative attractiveness and/or an increase in its relative number of goods. It should be noted, however, that changes in attractiveness and the number of goods are unlikely to offset the impact of labour force variations on prices and wages, as the former factors are typically independent of the latter.



Relations (7) – (9) are valid only at the full employment equilibrium. If, for any reason, price and wage adjustments are hindered, the adjustment occurs through changes in employment. The following section analyses the scenario where imperfections in the unskilled labour market prevent wage and price adjustments.

Equilibria with Imperfect Unskilled Labour Markets

Our aim is not to analyse the various types of labour market imperfections discussed in the literature. We simply assume that such imperfections exist and prevent adjustments in the unskilled labour market. We then study the impact of cross-country differences in labour force growth under these maladjustments. Therefore, we assume that, contrary to the skilled labour market, wages in the unskilled labour market do not adjust to supply and demand dynamics.

There are two ways to introduce maladjustment in the unskilled labour market. First, we could assume that the skill premiums in the countries, $w = w_H/w_L$ and/or $w^* = w_H^*/w_L^*$, are upwardly rigid—that is, they cannot exceed a certain level lower than the full employment skill premium. This describes a scenario where the nominal wage of unskilled workers is tied to that of skilled workers, to the average wage, or to the country's production prices. Consequently, such an imperfection straightforwardly leads to unskilled unemployment in the affected country or countries.

We will focus on a second market imperfection, which involves binding the unskilled wages of both countries. More specifically, we assume 1) that the relative unskilled wage $\varpi = w_L/w_L^*$ grows at a lower rate than its full employment rate $\bar{\gamma} = (n^* - n)/\sigma$, causing ϖ to increasingly deviate from its full employment value, and 2) that perfect competition prevails in the skilled labour market, ensuring full employment of skilled workers in both countries. As a result of assumption 1), the Foreign unskilled labour market faces constraints. This market imperfection is a more appropriate constraint because it aligns with the model's framework, which focuses on variations in cross-country prices and wages as the primary adjustment mechanism, and because it can depict a broader range of market misadjustments.

When all labour markets are perfectly competitive, full employment adjustment prevails. However, when wages are largely determined by institutional factors such as employer–employees bargaining, public policies, and labour legislation, assumption 1) implies that the countries' institutional decisions interact and jointly influence wage and price dynamics in each country, especially when exchange rate adjustments fail to offset changes in production costs expressed in national currencies.

Proposition 4. *Assume imperfections in the Foreign unskilled labour market that cause the relative unskilled wage $\varpi = w_L/w_L^*$ (i) to be lower than its full employment value and (ii) to grow at the rate $\tilde{\gamma} < \bar{\gamma} = (n^* - n)/\sigma$. Then, the slower growth of the Home labour force leads to increasing unskilled unemployment in the Foreign country.*

Proof See Appendix B.



Typically, slower labour force growth in the Home country should lead to a greater increase in its relative wages and prices. When market imperfections inhibit this increase, the slower growth of the Home labour force logically results in unemployment among Foreign unskilled workers, since it is the Foreign unskilled labour market that is constrained.

Income Transfers

Within our general equilibrium model, cross-country income transfers are a necessary condition for trade imbalances. Income transfers can encompass various mechanisms, the most common being financial flows. Since our model does not integrate capital accumulation and savings, we simply assume net income transfers from one country to another without specifying the source of these transfers. Several mechanisms may generate such transfers in a model without savings: public transfers (e.g. between countries in the European Union); ‘income mobility’ linked to personal mobility (e.g. German and French pensioners moving to Spain or Portugal); remittances associated with migration, etc.

Proposition 5. *A net income transfer φ from the Home country to the Foreign country results in:*

- 1) *A trade surplus (deficit) φ for the Home (Foreign) country.*
- 2) *A reduction in unemployment in the Foreign country and an increase in skill premia in both countries, provided that the Foreign country experiences unemployment of unskilled workers due to its unskilled wage being bound to the Home country’s unskilled wage.*

Proof See Appendix C. Note that if the Home country experiences unemployment among unskilled workers, the income transfer from Home to Foreign increases unemployment in the Home country.

Feature 1 of Proposition 5 is a straightforward result of the equilibrium. Feature 2 highlights that, in the presence of labour market imperfections, the increasing unemployment resulting from differences in labour force growth can be moderated and substituted by a trade deficit in the constrained (Foreign) country when income transfers occur. This effect is crucial when applying our results to explain the relationship between Germany and other European countries. It is important to note that the existence of a non-tradable sector is a prerequisite for the decrease in Foreign unemployment (see Appendix C).

Combining propositions 4 and 5 provides a general picture of how the adjustment can operate when the (unskilled) labour market is not perfectly competitive. A large range of equilibria are then possible, varying from balanced trade with no transfers and high unemployment to full employment with high transfers and trade imbalance.



Finally, it can be highlighted that the model behaves exactly as if there were only two country-specific goods (one Home good and one Foreign good), with the relative attractiveness being $\frac{\bar{a}}{\bar{a}^*} = \frac{a}{a^*} \left(\frac{N}{N^*} \right)^{1/\sigma}$.⁵ In other words, the model can be constructed to yield the same results by assuming two country-specific goods, where the countries' attractiveness, \bar{a} and \bar{a}^* , represents both the average appeal of the goods and the number of goods produced in each country. This feature will be utilised in the next section for constructing the extended model.

The Extended Model

Based on the stylised facts highlighted in the introduction, the model presented in the previous section is now used to analyse the relationships between Germany and other advanced economies. Nevertheless, to account for some developments which may have a sizeable impact on our outcomes, the model has been extended by incorporating new hypotheses.

First, the creation of the Eurozone has effectively eliminated one adjustment mechanism for the countries involved. Without exchange rate adjustments, variations in relative prices between Germany and other Eurozone countries that facilitate achieving full employment can only arise from wage adjustments. This can be difficult in imperfectly competitive labour markets. Moreover, the US dollar and currencies influenced by the dollar have been highly volatile relative to the German Deutsche Mark and the Euro, and these wide fluctuations are largely independent of inflation differentials. To account for these factors, we have firstly extended the model by introducing three advanced areas: Germany (*G*), Eurozone countries excluding Germany (*E*), and other advanced countries (labelled *N* for North). By considering three advanced economies, we can differentiate German goods from other goods in terms of elasticity of substitution, thereby revealing the impact of this difference on price and wage variations.

Secondly, many low-skill intensive production segments have been relocated to emerging countries since the early 1990s. Consequently, the price of each country-specific good depends not only on the production costs within the country itself but also on the costs in the emerging countries where those segments are offshored.⁶ This can alter the findings highlighted in the previous section, as the intensity of offshoring has varied significantly across advanced economies (Fig. 7). To model these impacts, we introduce two assumptions:

1) A new area designated as the South (*S*) is introduced, characterised by a large supply of unskilled labour with a low cost (wage). Thus, the World encompasses

⁵ Such an approach is presented in an early version of the model, see: Beissinger et al. (2020).

⁶ Harms et al. (2021) show that the decrease in the transportation cost linked to offshoring has a two-stage impact on domestic wages, firstly decreasing and subsequently increasing when the low transportation cost make the firms to locate each production step in the country with the related lowest cost.



four areas: three advanced regions (G , E and N) and the South, which is endowed solely with unskilled labour at wages significantly lower than those in advanced countries.

2) The production of tradable goods is divided in different segments, and those segments utilising unskilled labour can be offshored to the South, albeit with additional costs that vary across segments and countries.

These extensions, including three advanced countries, one emerging country, and the segmentation of production in the tradable sector, significantly increase the model's complexity. To limit this complexity, we have introduced two simplifying elements. First, based on the remark presented at the end of the preceding section, we assume only one country-specific tradable good in each advanced economy. Consequently, as noted above, the related coefficients in the utility function (a_G, a_E, a_N) now represent both the average attractiveness in demand and the number of goods of each country. Second, it is assumed that the South produces only non-traded services and the offshored segments of goods from G , E and N .

The Demand for Goods and Services

We extend the utility function (4) of the representative consumer by assuming three country-specific tradable goods denoted G , E and N :

$$u = y_{NT}^{1-\beta} y_T^\beta \text{ with } y_T = \left[a_G y_G^{\theta_1} + \left(a_E y_E^{\theta_2} + a_N y_N^{\theta_2} \right)^{\theta_1/\theta_2} \right]^{1/\theta_1} \quad (11)$$

where $\sigma_1 = (1 - \theta_1)^{-1}$ is the elasticity of substitution between country G 's tradable good and other tradable goods and $\sigma_2 = (1 - \theta_2)^{-1}$ denotes the elasticity of substitution between tradable goods E and N . The total demand functions for goods and services are shown in Online Appendix 2.1.

Production

The production of the tradable good Y_{Ti} , $i = G, E, N$, employs a Cobb–Douglas technology which combines (i) an intermediate good produced in a one-to-one ratio with high-skilled labour and (ii) F unskilled segments, each utilising one unit of low-skilled labour to produce one unit of the respective segment. The production function is:⁷

$$Y_{Ti} = A_i F^\alpha H_i^{1-\alpha} \prod_{f=1}^F L_{if}^{\alpha/F}, \quad i = G, E, N \quad (12)$$

⁷ Inserting F^α in the production function makes the cost of production and the prices of goods independent from the number of unskilled segments. This will permit to assume a continuum of stages.



where L_{if} denotes unskilled labour employed in the production of segment f in country i . Advanced countries can offshore some or all low-skill intensive segments to countries where their cost of production is lower.

Offshoring

Following Grossman & Rossi-Hansberg (2008), we model offshoring by comparing the offshoring cost with the offshoring gain (lower wage in the destination country). The cost of relocating production abroad consists of two components. First, irrespective of whether production is offshored to an advanced country or the South, the unskilled unit labour cost is increased by a fixed amount. This fixed amount accounts for the costs associated with transporting intermediate goods across countries and organising global production on an international scale. We assume for simplicity that this cost is sufficiently high to preclude offshoring across advanced countries, given the limited disparity in unskilled wages among them. Consequently, offshoring is only directed to the South.

Second, there is a cost which is specific to offshoring to the South and may vary across the three advanced economies. This variance reflects two main factors: (i) the potential deficiencies in infrastructures, workers' personal productivity, organisational capabilities, and enforcement of property rights etc. in emerging countries; and (ii) the dependency of the relocation cost on the geographic, cultural, and historical ties each advanced country maintains with certain emerging countries. Given that the impact of these factors on production costs can vary significantly across production segments, we assume that for each country i , the cost of relocating and producing segment f in the South at time t , ω_{ft}^i , differs across segments $1, \dots, F$. Ordering the segments by increasing offshoring cost, the cost ω_{ft}^i producing segment f if good i in the South is defined by:

$$\omega_{ft}^i = w_{Lt}^S \times (\kappa_{it})^{f/F}, \quad \kappa_{it} > 1, f = 1, \dots, F, i = G, E, N, \quad (13)$$

where w_{Lt}^S is the unit wage of low-skilled labour in the South including the fixed amount representing the common offshoring cost, and $(\kappa_{it})^{f/F} > 1$ is the multiplicative factor determining the extra cost of producing the segment f of good i in the South.

Let K_{it} be the segment where the production cost is equal in both advanced country i and the South at time t . Hence, K_{it} is the number of offshored segments in the production of good $i = G, E, N$, and $k_{it} = K_{it}/F$ its proportion. Consequently, the proportion of segments remaining in country i is $1 - k_{it}$. We can write (time index t is omitted for simplicity):⁸

$$\kappa_i = (w_L^i / w_L^S)^{1/k_i} \quad (14)$$

⁸ By definition of K_i : $\omega_{K_i} = w_L^S \times \kappa^{k_i} = w_L^i \Rightarrow \kappa = (w_L^i / w_L^S)^{1/k_i} \Leftrightarrow k_i = \log(w_L^i / w_L^S) / \log \kappa$



Consider country $i = G, E, N$, which offshores the proportion k_i of unskilled segments in the production of good i . Assuming a continuum of low-skilled production segments, the price of the tradable good i is (proof in Online Appendix 2.2):

$$p_i = \frac{(w_H^i)^{1-\alpha} (w_L^S)^{\alpha k_i/2} (w_L^i)^{\alpha(1-k_i/2)}}{A_i \alpha^\alpha (1-\alpha)^{1-\alpha}} \quad (15)$$

Factor Demands and General Equilibrium

Table 1 depicts the system of equations that defines the general equilibrium of the extended model used for simulations, as detailed in Online Appendix 2. We summarise below the successive steps involved in generating this system.

Firstly, the maximisation of utility and profit allows us to define the demands for goods and factors in each country in relation to the world's total income $I_W = I_G + I_E + I_N + I_S$. From the South's balanced trade, we subsequently determine the South's income I_S and thereby the world income I_W in relation to the income of advanced countries ($I_G + I_E + I_N$).

By equating supply and demand, we obtain the equilibrium conditions in the labour markets which determine relative wages as functions of relative labour supply for each advanced country. These labour market equilibrium relations are combined with the price and offshoring equations to generate 13 equations with 13 unknown variables, defining the full employment general equilibrium.

Based on this equilibrium, we calculate (i) the consumer prices P_i , the real wages w_L^i/P_i and w_H^i/P_i , and the real income per capita $(I_i/P_i)/(\bar{L}_i + \bar{H}_i)$ for each country $i = G, E, N$, and (ii) the relative production prices p_G/p_j and the relative consumer prices P_G/P_j , $j = E, N$.

In the case of imperfections in unskilled labour markets, the system comprises as many additional equations and endogenous variables as the number of imperfect markets. The additional equations define the unskilled wage indexations, and the additional unknown variables are the constrained L_i s. With L_i determined at the model equilibrium, the unemployment rate of unskilled workers $u_L^i = (\bar{L}_i - L_i)/\bar{L}_i$ can be calculated. Regarding income transfers, they are incorporated into the demand functions for goods and factors.

Simulations

We now simulate the extended CGE model developed in the preceding section. The simulations are based on values of the parameters and of the exogenous variables that align with the model's structure and are calculated from the observed economic developments in each region (Germany, Eurozone, and North). In “[The Three Simulations](#)” sub-section, we define the three specifications selected for the simulations. “[Variables and Parameters](#)” sub-section details the empirical determination of the



Table 1 The general equilibrium equations^a

Exogenous parameters:	$\alpha, \beta, A_G, A_E, A_N, a_G, a_E, a_N,$
Exogenous variables:	$k_G, k_E, k_N, w_L^S, \bar{L}_G, \bar{L}_E, \bar{L}_N, \bar{H}_G, \bar{H}_E, \bar{H}_N,$ $w_H^G = 1.$
13 endogenous variables:	$w_L^G, w_L^E, w_L^N, w_H^E, w_H^N, p_G, p_E, p_N, P_{EN}, P_T,$ I_G, I_E, I_N
13 equations ($i = G, E, N$):	
$p_i = A_i^{-1} (w_H^i / (1 - \alpha))^{1-\alpha} \alpha^{-\alpha} (w_L^S)^{\alpha k_i / 2} (w_L^i)^{\alpha(1-k_i/2)}$	(3 equations)
$P_{EN} = \left(a_E^{\sigma_2} P_E^{1-\sigma_2} + p_N^{1-\sigma_2} \right)^{\frac{1}{1-\sigma_2}}$	(1 equation)
$P_T = \left(a_G^{\sigma_1} p_G^{1-\sigma_1} + P_{EN}^{1-\sigma_1} \right)^{\frac{1}{1-\sigma_1}}$	(1 equation)
$I_i = w_L^i \bar{L}_i + w_H^i \bar{H}_i$	(3 equations)
$w_H^i = \frac{\beta(1-\alpha)}{(1-\beta)(1-\alpha) + (1-k_i)\alpha} \frac{\bar{L}_i}{\bar{H}_i} w_L^i$	(3 equations)
$\bar{H}_G = \frac{a_G^{\sigma_1}}{w_H^G} \left(\frac{P_{EN}}{p_G} \right)^{\sigma_1-1} \frac{(1-\alpha)\beta(I_G + I_E + I_N)}{\left(\frac{p_T}{P_{EN}} \right)^{1-\sigma_1} - \alpha \left(\frac{k_G a_G^{\sigma_1}}{(p_G/P_{EN})^{\sigma_1-1}} + \sum_{i=E,N} \frac{k_i a_i^{\sigma_2}}{(p_i/P_{EN})^{\sigma_2-1}} \right)}$	(1 equation)
$\bar{H}_E = \frac{a_E^{\sigma_2}}{w_H^E} \left(\frac{P_{EN}}{p_E} \right)^{\sigma_2-1} \frac{(1-\alpha)\beta(I_G + I_E + I_N)}{\left(\frac{p_T}{P_{EN}} \right)^{1-\sigma_1} - \alpha \left(\frac{k_G a_G^{\sigma_1}}{(p_G/P_{EN})^{\sigma_1-1}} + \sum_{i=E,N} \frac{k_i a_i^{\sigma_2}}{(p_i/P_{EN})^{\sigma_2-1}} \right)}$	(1 equation)
Additional related variables:	$P_i = (w_L^i)^{1-\beta} (P_T)^\beta p_G / p_i, P_G / P_j$ $w_i = w_L^i / w_H^i, w_L^G / w_L^j, w_L^i / P_i, w_H^i / P_i,$ $i_i = (I_i / P_i) / (\bar{L}_i + \bar{H}_i), i = G, E, N, j = E, N.$

^aThe equilibrium equation on the market for skilled labour in North H_N is deleted because of Walras' Law and the wage of skilled workers in Germany is selected as numeraire.

values of the parameters and exogenous variables used in the simulations. “[Results](#)” sub-section presents and interprets the simulations results.

The Three Simulations

We conduct three simulations, each covering the period 1975–2015. The first simulation explores a variant of the extended model featuring no offshoring, competitive labour markets, and balanced trade. This simulation aims to assess the cross-regional dynamics of wages and prices caused by divergent labour force growth rates that would be in line with full employment and balanced trade in all countries. We also highlight the model predictions regarding unemployment and trade balance when wages and prices adjust in accordance with observed facts, thus deviating from full employment conditions.

The second simulation incorporates all the extensions discussed in “[A stylised Model](#)” and “[The Extended Model](#)” sections: pegged unskilled wages between the Eurozone and Germany (reflecting imperfections in unskilled labour markets), income transfers between regions, and offshoring to the South, which can vary across regions. Although Simulation 2 represents a more realistic framework than



Simulation 1, it does not aim to depict actual economic developments, as it excludes a wide range of events with diverse impacts on each region, such as oil shocks, technological changes, geopolitical changes, financial crises, and the euro area debt crisis. Instead, the simulation seeks to assess the effects of the factors analysed in our propositions (divergent labour force growth rates with imperfect unskilled labour markets and income transfers) as well as the impact of offshoring. This assessment employs values of the parameters and exogenous variables calculated from available databases.

Finally, we simulate a counterfactual scenario (Simulation 3) that replicates all the characteristics of Simulation 2, except for the differences in labour force growth, which are assumed to be zero across the three areas. This counterfactual exercise will be compared with the second simulation to quantify the specific contribution of the divergence in labour force growth rates to the simulated outcomes.

Table 2 summarises the setup of the three simulations.

Variables and Parameters

We give a brief overview of the way we have selected the data and constructed the series utilised in the simulations concerning (i) the labour force in each area (Germany, Eurozone and North), (ii) the introduction of labour market imperfections, (iii) the income transfers across countries, (iv) the offshoring dynamics, (v) the elasticities of substitution and (vi) the other parameters and exogenous variables. A detailed presentation and justification of those choices and measurements are available in Online Appendix 3.

Labour Force

The labour forces and their variations are measured annually in terms of working hours, as depicted in Fig. 1c. This is important because countries and firms have, to different degrees, utilised the tools of working time reductions and working time accounts to mitigate or avoid unemployment.

The impact of the unification-related surge in the German labour force has been smoothed over ten years to account for the policies taken by the German governments to prevent the increase in unemployment in the transition period following

Table 2 The simulation characteristics

	Simulation 1 Basic model	Simulation 2 extended model	Simulation 3 counterfactual
Differences in labour force size	Yes	Yes	Yes
Differences in factor endowments	Yes	Yes	Yes
Differences in labour force growth	Yes	Yes	No
Offshoring to the South	No	Yes	Yes
Unskilled labour market imperfection	No	Yes	Yes
Income transfers	No	Yes	Yes



reunification. Additionally, we have excluded the 1990–1995 period from the simulation, as the model is evidently inadequate for this time frame due to its assumption of a ‘unified’ German structure, which did not exist in this period. We account for differences in skill endowments between areas; however, we do not consider the increase in the skill level across the three areas from 1975 to 2015. A discussion of these assumptions and a detailed presentation of the data and methods used for labour force calculations can be found in Online Appendix 3.1.

Labour Market Imperfections

In accordance with the theoretical approach, imperfections in the unskilled labour markets are introduced in Simulations 2 and 3 by pegging wages between Germany and the Eurozone. This mechanism can lead to unemployment in these areas. Pegged wages are implemented by incorporating into the simulations the observed ratio of labour cost per hour between Germany and the Eurozone, measured in the same currency. Given that the model’s structure assumes identical variations in labour productivity across the two areas, we have neutralised the effect of differences in labour productivity growth by considering the following productivity-adjusted ratio of unit labour costs:

$$\eta_{G/E} = \frac{\text{labour share of total income (in\% in Germany)} \times \frac{\text{GDP in current US dollar in Germany}}{\text{employment in hours in Germany}}}{\text{labour share of total income (in\% in Eurozone)} \times \frac{\text{GDP in current US dollar in Eurozone}}{\text{employment in hours in Eurozone}}}$$

To account for the lag between changes in unit labour costs and subsequent changes in prices, demand, and production, we have applied the average of the pegged wages from years $(t-1)$ and t in the simulation for year t . Details on the data, methods, and a discussion of the constraints and potential biases associated with the selected data are provided in Online Appendix 3.2.

We did not assume pegged wages between the North and the other areas because, within a general equilibrium model with price adjustments, the large volatility of the US dollar would have resulted in large, rapid, and unrealistic fluctuations in unemployment (see the discussion in Online Appendix 3.2). Since pegging unit labour costs between Germany and the North is untenable, we introduced North’s employment (labour force minus unemployment) as an exogenous variable in Simulations 2 and 3, making the unskilled wage of the North endogenous. Consequently, the model no longer determines unemployment in the North, and the simulations focus solely on the differences between Germany and the Eurozone.

Income Transfers

Inter-area income transfers have been incorporated by calculating the current account balances between Germany and the other two regions. Using data from the



Deutsche Bundesbank, we have calculated the annual ratio of the current account balance between Germany and each region j ($j = E, N$) relative to German GDP.⁹ In the simulations, these ratios have been applied to Germany's total income (the sum of factor incomes), and the corresponding amounts have been transferred to the respective region's total income. These ratios are illustrated in Fig. 6 ("Stylised Facts and Literature" section).

Offshoring

In the model, k_{it} denotes the proportion of low-skilled segments offshored to the South in the production of tradable good i ($i = G, E, N$) in period t . We quantify k_{it} by calculating:

$$k_{it} = \frac{\psi_{Si,t}}{\psi_{Si,t} + \psi_{ii,t}}, \quad i = G, E, N \quad (16)$$

where $\psi_{Si,t}$ and $\psi_{ii,t}$ respectively denote the value-added contribution of Southern and of country i 's unskilled workers to the final production of tradable good i in period t .

Note that $\psi_{Si,t}$ encompasses not only the low-skilled value added embedded in direct flows of intermediate goods from the South to region i , but also the indirect flows of Southern intermediate goods embedded in the imports of region i from other advanced regions. In other words, the calculation of $\psi_{Si,t}$ accurately reflects the global value chains linking the South and region i .

Online Appendix 3.3 lists the countries included in Eurozone, North, and South and provides details on data selection and construction. We calculate k_{it} using the 2013 release of the World Input-Output Database (WIOD), as described by Timmer et al. (2015). For Germany and each country belonging to Eurozone and North, an industry is classified as tradable (resp. non-tradable) if its gross exports lie above (resp. below) the 25th percentile of the gross exports' distribution in this country in 1995. Consequently, the tradable sector in a country comprises both manufacturing and service sectors, and its composition may differ across countries. As a robustness check, we compare this first definition with the definition considering manufacturing as the tradable sector. As shown in Online Appendix 3.4, for the three regions (Germany, Eurozone, North) the k_{it} values for the tradable sector (first definition) are on average about 10 percentage points lower than for manufacturing in the period 1995-2009, but both time series display similar dynamics over time. We use the first definition of the tradable sector in our simulations because it includes tradable services. For the period 1995-2009, the average value of k_i for the tradable sector is 0.40 for Germany, 0.29 for Eurozone, and 0.26 for North. Online Appendix 3.4 (i) provides further details on the calculation of the k_{it} s, (ii) compares these values for the tradable sector and the manufacturing sector, and (iii) presents the method used to calculate k_{it} for the missing years.

⁹ <https://www.bundesbank.de/dynamic/action/en/statistics/time-series-databases/time-series-databases/743796/743796?treeAnchor=AUSSENWIRTSCHAFT&openNodeId=1300618>.



Table 3 The model parameters and the South unskilled workers' wage

α	β	A_G, A_E, A_N	a_G	a_E	a_N	w_L^S
0.3	0.75	1	0.1232	0.318	1.0	0.08

Calculation of the Elasticities of Substitution σ_1 and σ_2

The elasticities of substitution, σ_1 and σ_2 , are determined in three steps: (i) theoretical derivation, (ii) econometric estimation, and (iii) validation against previous literature.

1) *Theoretical mapping.* Based on our theoretical model, we express σ_1 and σ_2 as functions of the price elasticities of exports for Germany, Eurozone, and North (ε_i , $i = G, E, N$). At this level of aggregation, changes in a region's tradable price affect the world tradable goods price, which we account for in our derivation (see Online Appendix 3.5.1 for details). The elasticities of substitution so determined are:

$$\sigma_1 = \frac{\varepsilon_G}{1 - S_G}; \sigma_2 = \frac{1}{1 - (S_j/(1 - S_G))} \left(\varepsilon_j - \sigma_1 \frac{S_G}{1 - S_G} S_j \right), j = E, N$$

where S_i depicts region i 's share of the world demand for tradable goods: $S_i \equiv \frac{p_i Y_i^d}{\beta I_W}$.

Note that (i) σ_2 can be identified based on the price elasticity of exports and the expenditure share of either Eurozone, or North; and (ii) σ_2 depends not only on ε_j and S_j (where $j = E$ or N), but also on the market share of German tradable goods and on the price elasticity of German exports (since σ_1 enters into the determination of σ_2).

2) *Empirical estimation.* We estimate the price elasticities of exports ε_i , $i = G, E, N$, using annual sectoral data from the 2013 release of the World Input-Output Database (WIOD), employing fixed-effects panel regressions for the period 1995–2007. The WIOD allows for a consistent calculation of real exports, real effective exchange rates, and real foreign demand for the manufacturing sectors in the three regions (see Online Appendix 3.5.2 for data selection and construction). The panel estimation approach (see Online Appendix 3.5.3) includes sectoral fixed effects to account for time-invariant heterogeneity across sectors. To assess robustness, we estimate multiple model specifications, including both weighted and unweighted regressions, as well as versions with linear time trends or year dummies (see Online Appendix 3.5.3, Tables 2–7). To avoid overfitting, the set of explanatory variables is restricted to those with strong theoretical justification. Model selection is guided by the Akaike Information Criterion (AIC) and the adjusted R^2 , with models featuring a linear trend or year dummies best reflecting the data-generating process. In our preferred specifications, the absolute value of the estimated price elasticity of export demand ranges between 0.55 and 0.57 for Germany, between 0.59 and 0.66 for Eurozone, and between 0.44 and 0.64 for North. As discussed in the Online Appendix, we use $\hat{\sigma}_1 = 0.64$ and $\hat{\sigma}_2 = 0.87$ as intermediate values for our simulations.

Notably, in all estimated model variants with time fixed effects or with a deterministic trend, the elasticity of substitution between German goods and other goods



is consistently lower than that between Eurozone and North ($\sigma_1 < \sigma_2$). This systematically magnifies the impact of demographic differences, as outlined in Proposition 1.

3) *Literature validation.* To validate our empirical estimates, we compare them with findings from the literature for Germany and the euro area for comparable time periods and levels of aggregation (using annual or quarterly data). Our estimates for the price elasticity of exports for the euro area are of a similar order of magnitude as those reported in previous studies. The estimation results for Germany fall within the middle range of export price elasticities found in the literature. (see Online Appendix 3.5.4 for details). Estimates for earlier decades (the 1970s and 1980s) indicate that export demand elasticities have been of a similar order of magnitude during those periods as well.

Other Parameters and Exogenous Variables

Table 3 shows the parameters and exogenous variables utilised in the three simulations.

The value of α indicates that 30% of the total labour income is allocated to less skilled workers in the tradable sector, which aligns with what was observed on average in advanced economies during the late 1970s and throughout the 1980s. The value of β implies that non-tradable, unskilled-intensive services account for 25% of total expenditure, which is consistent with observations in the economies considered.

We already explained that parameters a_i , $i = G, E, N$, depict both the areas' attractiveness in demand and the number of differentiated goods supplied by each area. Although there are no data available to directly approximate these parameters, we have qualitative information about the developments affecting them over the period from 1975 to 2015. Consequently, we first select coefficients that replicate the observed unemployment gap between Germany, the Eurozone, and the North in 1975 within Simulation 2 (which includes imperfect labour markets, income transfers, and changing offshoring for each area). We then maintain these coefficients in the simulations throughout the period from 1975 to 2015. By comparing the so-calculated unemployment gap in Simulation 2 with the observed unemployment gap, we determine the changes in ratio a_E/a_G that align the simulations with observed reality and assess whether these variations are consistent with the available information.

The wage in the South w_L^S , which includes the fixed offshoring cost but excludes the variable costs associated with each production segment, constitutes between one-third and one-fourth of the unskilled wage in the three advanced areas. The total factor productivities (TFPs) have been chosen to be identical and equal to 1 in the production of tradable goods across the three advanced areas. Because the results of the simulated model are expressed in terms of relative changes rather than absolute levels, this setup is equivalent to assuming the same rate of TFP growth in all areas. However, the discrepancy in TFP growth between Germany and the Eurozone (notably, several Southern European countries have experienced higher TFP growth



than Germany since 1975) is accounted for in the labour cost pegging (see “[Labour Market Imperfections](#)” section).

Results

We focus solely on the results and the comparison of simulated versus observed data for Germany and the Eurozone, as detailed in “[Labour Market Imperfections](#)” section and Online Appendices 3.1 and 3.2.

In a first step, we compare the variations in relative wages and prices as determined by Simulation 1 (characterised by differences in labour force growth, competitive labour markets, absence of income transfers, and no offshoring) with observed data. We highlight the disparities in wage and price variations between Simulation 1 and the observed developments, discussing their implications in terms of market adjustment, unemployment and trade balance as predicted by the model.

In a second step, we compare the outcomes of Simulations 2 and 3 to evaluate *ceteris paribus* the contribution of the differences in labour force growth to observed developments. Given that the Eurozone/Germany relative unskilled wage, the current account balances, and the employed labour force of the North are exogenously integrated into Simulation 2 (see “[Variables and Parameters](#)” section and Online Appendix 3.2), the comparison will concentrate on the unemployment gap between the Eurozone and Germany as the key endogenous variable determined by the model.

Finally, we compare the outcomes of Simulation 2 with observed data. As previously mentioned, Simulation 2 cannot perfectly replicate observed developments because various shocks (e.g. oil shocks, financial crises) are not considered, and the model underlying Simulation 2 does not capture the entire complexity of the mechanisms influencing the economic situation of the three regions. We therefore highlight the omitted variables and missing mechanisms that may account for the divergence between Simulation 2 and the observed data.

In the forthcoming figures, the non-simulated years 1990–1995 (as explained in “[Labour Force](#)” section) are depicted by a shaded area.

Simulation 1

Figure 8a contrasts the relative wage and price ratios (Germany/Eurozone) obtained from Simulation 1 (solid line) with the observed counterparts (dashed line). Both series are presented as index values with 1975 as the base year.¹⁰ An increase in the simulated values indicates that German wages and prices should have risen relative to those in the Eurozone to sustain full employment and balanced trade.

¹⁰ In Simulation 1, relative prices and wages (Germany/Eurozone) have identical variations. This is because, without offshoring and with perfect competition, balanced trade and given skill endowments, both skilled and unskilled labour wages have identical growth rates as well as prices which are combinations of both wages. We do not display the comparison of the relative real income per capita (available from the authors) because this would lead to the same diagnosis.



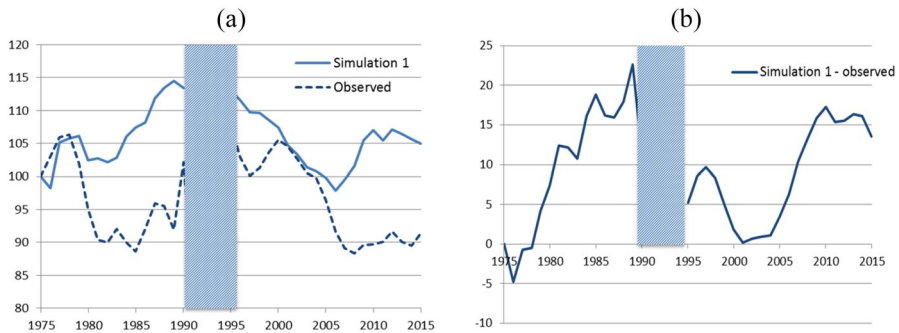


Fig. 8 Relative wages and production prices (Germany/Eurozone, 1975=100). *Note:* The shaded surface covers the non-simulated years 1990–1995. (a) draws the variation in the relative (Germany/Eurozone) wages and prices corresponding to the basic model (solid line; Simulation 1 with no offshoring, no transfers and competitive labour markets) compared to the observed variation in wages (dashed line). (b) draws the difference between the simulated variations (Simulation 1) and the observed variations

Figure 8b illustrates the difference between the simulated and observed values of German wages and prices relative to other countries. An increase in this differential suggests that German wages and prices have risen too slowly compared to those in other countries given the variations in labour forces, which leads to the prediction of an increase in Eurozone's unemployment in relation to Germany (Proposition 2) and/or an income transfer from Germany to Eurozone, generating a growing German current account surplus (Proposition 3). A reduction of this differential leads to the opposite prediction (decrease in Eurozone relative unemployment or/and decrease in the German current account surplus).

According to Fig. 8, three periods can be distinguished:

1. *Late 70s to reunification:* During this period, the actual German/Eurozone relative wages and prices decreased, whereas the simulated ones increased by 15%. In this case, the model predicts that unemployment should have increased in Eurozone compared to Germany and/or the German current account surplus should have risen.
2. *1995 to 2005:* From 1995 to 2002, the difference between the simulated and observed wages and prices steadily narrowed, and the simulated and observed values broadly coincided from 2002 to 2005. Our model consequently predicts that the unemployment gap (Eurozone–Germany) should have decreased and/or the German current surplus should have declined.
3. *2005 to 2015:* From 2005 to 2010, the disparity between the simulated and observed values of the German/Eurozone relative wages and prices widened again, and this divergence remained roughly constant from 2010 to 2015. Here, the model predicts that the unemployment gap (Eurozone–Germany) and/or the German current account surplus should have increased, with a stabilisation in the early 2010s.

The observed developments of the unemployment gap (Eurozone–Germany) and of the German current account depicted in “[Stylised Facts and Literature](#)” section (Figs. 5 and 6) confirm the model predictions.

Simulations 2 and 3

Simulation 2 combines differences in labour force growth rates with offshoring, labour market imperfections, and income transfers. The counterfactual Simulation 3 replicates the same developments except the differences in labour force growth. By comparing Simulations 2 and 3, we can assess the *ceteris paribus* impact of the divergence in labour force growth rates on the model outcomes.

Figure 9 illustrates the unemployment gap (Eurozone–Germany) as calculated by Simulation 2 (dashed line) as well as the difference in the unemployment gap

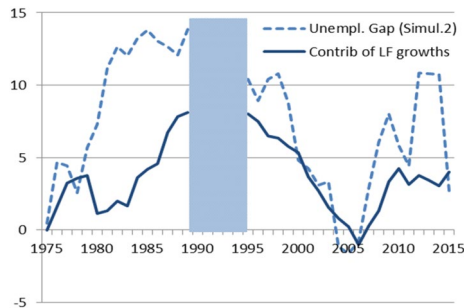


Fig. 9 Contribution of the difference in labour force growth across all three areas to the unemployment gap (Eurozone–Germany). *Note:* The unemployment gap (dashed line) is the difference between the Eurozone (except Germany) unemployment rate and the German unemployment rate calculated by Simulation 2 (extended model). The solid line draws the contribution of the between-area differences in labour force growth to this unemployment gap, calculated by the difference between the unemployment gap in Simulation 2 (extended model) and the unemployment gap in Simulation 3 (extended model without differences in labour force growth)

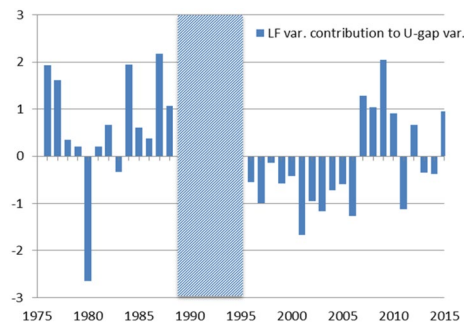


Fig. 10 Annual contribution of the difference in labour force growth to the variation of the unemployment gap (Eurozone–Germany). *Note:* Difference between the annual change in the unemployment gap (Eurozone–Germany) in the Simulation 2 (extended model) and the annual change in the unemployment gap in Simulation 3 (extended model without differences in labour force growth)



between Simulations 2 and 3 (solid line). The latter is interpreted as the contribution of the differences in labour force growth across the three regions to the unemployment gap (Eurozone–Germany) as predicted by the model. Figure 10 depicts the contribution of the difference in labour force growth across regions to the annual change in the unemployment gap (Eurozone–Germany). This contribution is calculated as the difference between the annual change in the unemployment gap from Simulation 2 and that from Simulation 3.

Figures 9 and 10 demonstrate that, according to our extended model—which incorporates offshoring, labour market imperfections, and income transfers—the differences in labour force growth across regions were a key driver of the increase in the unemployment differential (Eurozone–Germany) from 1980 to 1990, of its decrease from 1995 to 2006, and of its subsequent rise from 2006 to 2010. Over the periods 1975–1989 and 1996–2015, differences in labour force growth account for, on average, approximately 50% of the unemployment gap between Eurozone and Germany.

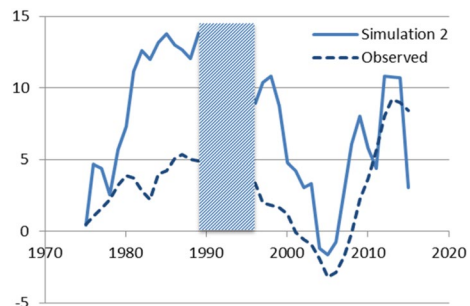
Simulation 2 Versus Observed Facts

Figure 11 compares the unemployment gap (Eurozone–Germany) calculated by Simulation 2 with its observed level. Although the two series are not congruent, the general trends and turning points of the simulated data align well with those observed.

Regarding the difference between the simulated and observed unemployment gap (Eurozone–Germany), several distinct periods can be identified. First, from the late 1970s up to 1990, the simulated increase in the unemployment gap is significantly greater than the observed increase. Second, from the mid-1990s to the mid-2000s, the difference between the simulated and the observed unemployment gaps sharply decreased, from about 7% to about 1.5%. Subsequently, the difference fluctuated, showing a substantial positive divergence during the 2008–2009 financial crisis and a significant negative divergence in 2015.

As noted in “[Other Parameters and Exogenous Variables](#)” sub-section, the simulations were conducted under the assumptions of no change in the attractiveness and in the number of country-specific tradable goods. From our model, we have calculated that, to replicate the observed change in the unemployment gap between

Fig. 11 Unemployment gap (Eurozone–Germany): simulated vs. observed data



Germany and Eurozone, the ratio a_E/a_G should have increased by 8.7% from 1975 to 1990 and decreased by 4.8% from 1995 to 2004. These variations align with the structural changes within the European Union during these periods. The 1970s and 1980s were characterised by a catching-up process in several Southern European countries (Greece, Portugal, Spain), typically corresponding to an enhancement in attractiveness and an increase in the number of the goods produced. This reflects both the inflow of FDI from more advanced countries and an adaptation to global demand, which increase the a_E/a_G ratio. Conversely, from the mid-1990s, multinationals relocated several productions from Southern Europe to Central and Eastern Europe, thereby reducing the attractiveness and number of goods of those countries. This typically results in a decrease in a_E .

Finally, several additional factors can explain the difference between the simulated and observed unemployment gap:

1. Our model disregards the existence of a public sector, which tends to increase simulated unemployment compared to observed unemployment, as the public sector can absorb workers made redundant in other sectors. The public sector constitutes between 20 and 30% of total employment in Germany and Eurozone countries, and this share was higher in the Eurozone than in Germany during the period 1975–1990. Notably, the share of public employment in total employment in France (representing 30% of the Eurozone labour force) rose significantly, from 23% in the mid-1970s to 30% in the early 1990s. In contrast, this share was only 21% in Germany by 1991.¹¹ Overall, public employment played a comparatively greater role in reducing unemployment in the Eurozone than in Germany, especially before reunification.
2. As mentioned in “[Labour Force](#)” sub-section, we assumed no change in the regions’ skill endowments. This assumption could have led to higher simulated unemployment in the Eurozone in the late 1970s and 1980s compared to observed unemployment, as these periods were characterised by a substantial increase in skill endowments in Southern Europe.
3. We have observed that the difference between the simulated and observed unemployment gaps (Eurozone–Germany) noticeably increased during the 2008–2009 financial crisis and became significantly negative in 2015. The increase in 2008–2009 reflects the specific strategy of German firms to retain employees by drastically reducing working hours (Burda & Hunt 2011), which prevented a surge in German unemployment, unlike in other countries. The difference in 2015 reveals that the observed unemployment gap was significantly higher than the simulated gap (see Fig. 11). The euro debt crisis and the very restrictive policies implemented in Southern European countries during the 2010s dramatically raised unemployment. Additionally, the wage compression in Southern Europe was insufficient to counteract this trend, whereas in the simulations—which do

¹¹ We focus on the case of France because it is the only country whose variation in public employment substantially differs from that of Germany.



not account for the critical decrease in public expenditure—their positive impact prevails.

Discussion and Conclusion

The model developed in this paper highlights the impact of cross-country divergence in labour force growth on relative wages and prices in open economies. It demonstrates that a country with a structurally low increase in its labour force will experience higher growth in wages and prices compared to its trade partners if full employment and balanced trade prevail in all countries. Otherwise, trade partners may face unemployment and/or trade deficits. This can generate severe constraints on the trade partners' policy options, particularly when the country in question pursues a policy of wage moderation.

The simulations, conducted using parameters calculated from the World Input-Output Database and exogenous variables observed in three groups of countries (Germany, the Eurozone and the North), indicate that this 'demographic channel' is consistent with the German experience and its relationship with other advanced economies since the mid-1970s. Furthermore, our estimates suggest that the elasticity of substitution between German goods and those from the other two regions is lower than the elasticity of substitution among goods produced within those regions. This amplifies the impact of the demographic channel on wages and prices.

During periods when Germany experienced lower labour force growth than its trade partners without a corresponding increase in German prices and wages (1975-1990 and 2005-2013), those trade partners typically faced higher unemployment and/or a substantial trade deficit with Germany. Conversely, in the 1995-2005 period, which followed the significant increase in the German working population due to reunification, the maintenance of relatively high wages in Germany resulted in a substantial rise in German unemployment as well as a significant decrease in its trade surplus.

Consequently, our model offers an additional explanation for the economic turmoil Germany experienced between 1995 and 2005. The so-called German 'sickness' was not solely a consequence of low labour market flexibility, as frequently suggested in the literature. After all, the German economy had previously thrived despite relatively inflexible labour market institutions prior to 1990. Our model provides a new argument for the conventional diagnosis that the German disorder stemmed from the conflict between the changes brought about by reunification and the existing labour market structure. Reunification led to a substantial increase in the labour force without a corresponding rise in the number and attractiveness of German goods. As a result, Germany would have needed to reduce wages and prices to maintain full employment, which was at odds with the prevailing labour market structures.

The changes in labour market institutions implemented in the late 1990s (Dustman et al. 2014) and the Hartz reforms of 2003–2005 permitted to adapt the labour market institutions to the new conditions imposed by reunification. It is important to highlight that, in addition to reunification, the extensive offshoring undertaken



by German firms from the early 1990s constituted an additional shock to the German labour market. This was largely addressed by expanding a non-tradable service sector characterised by low labour costs and part time jobs, and the reforms implemented in the late nineties and early 2000s have permitted this expansion by liberalising the German labour market. (A precise presentation of those reforms as well as a discussion on their impacts on the German labour market can be found in Beissinger et al., 2016, “[Related Literature: Demographic Changes and German Economic Performance](#)” section, and 2.3.) This expansion resulted in labour market polarisation between stable, well-paid positions (in manufacturing) and short-term contracts involving low pay and part-time work (Beissinger et al. 2016). From 2005 onward, the persistent low growth in the German labour force re-established the need for higher wage and price growth in Germany. However, these increases did not materialise. The downward pressure on German wages, originating in the low-skill non-tradable services sector, eventually spread to other sectors, thereby reducing production costs and inflation. In summary, the labour market reforms implemented in the late nineties and early 2000s mitigated the effects of reunification on German unemployment, but they also increased wage pressure and contributed to higher unemployment in other Eurozone countries from the mid-2000s onward, once the effects of low labour force growth in Germany started to be felt again.

From 2010 onwards, due to the restrictive policies imposed on Southern European countries during the Euro debt crisis, their primary avenue to stimulate economic growth was to expand their external market shares, which in the short term could only be achieved by enhancing price-competitiveness. The modest increase in German prices and wages, coupled with the relatively low growth of the German labour force compared to Southern European countries in the 1990s and the 2000s, made this strategy challenging to implement. Between 1991 and 2010, the German labour force grew by only 3.2%, compared to 50.6% in Spain, 28% in Greece and 15% in Portugal.¹² Consequently, as shown in Figure 9, the disparity in labour force growth combined with German labour market reforms likely played a key role in the challenges faced by Southern European countries in formulating their policies and in the severe economic crises they experienced from 2010 to 2015.

Could the labour force channel highlighted here remain a key element in the relationship between Germany and other euro area members in the forthcoming years? The answer to this question hinges on the between-country differences in labour force growth. Over the last two decades, Southern European countries have experienced a prolonged and substantial decline in their natural demographic growth, resulting in their labour force now growing more slowly than Germany's. Conversely, other Eurozone countries, such as France, Ireland, and Belgium, continue to exhibit labour force growth that surpasses Germany's. Our model indicates that, to avoid unemployment or trade deficits, these countries must either sustain an increase in wages and prices that remains permanently lower than Germany's or achieve a

¹² Labour force in persons, OECD Stat. The substantial increase in Spain is essentially due to the large inflow of migrants in this period.



lasting increase in the attractiveness and number of goods produced relative to Germany (Propositions 2 and 3).

However, recent years have seen a shift in Germany's economic dynamics, with higher wage growth and increased inflation rates. This development could alter the constraints previously imposed on Germany's euro area partners, potentially providing more flexibility in their wage-setting policies. The relaxation of these constraints might help balance the economic relationships within the Eurozone, but it also raises the question of how Germany can maintain its competitiveness.

To address these challenges, Germany could consider enhancing its labour force growth through measures such as increasing the average number of hours worked and encouraging immigration. Given that a proactive pro-birth policy would only be effective for the next working generation, these measures could provide more immediate solutions. However, the impact of immigration also depends on the type of immigrants, skilled or unskilled. If migration policy primarily attracts high-skilled workers, the positive effect of the increase in the labour force is strengthened by the increase in the German skill endowment. This tends to raise the unskilled wage in Germany, thereby reducing the pressure on unskilled wages (and employment) in other Eurozone countries.

Finally, our model highlights the strong interdependence between countries within the Eurozone. With a common currency, wage adjustments become the primary means for relative price adjustment between countries, and differences in labour force growth can therefore have destabilising effects. This underscores the potential need for greater European fiscal integration. Fiscal federalism, i.e. a common budget with automatic stabilisers, may help mutualise the burden of adjustment and mitigate the risks to employment and external balances that currently fall on individual member countries.

Appendix A: Determination of Relations (7)-(9)

Households and demand

The utility function (4) described in “[Demand for Goods and Services](#)” section is maximised subject to the consumers' budget constraint. After aggregation (each non-tradable service is only sold in its country whereas all tradable goods are purchased in both countries), we determine the total demand (superscript d) for each non-tradable service (NT and NT^*) and for the Home and the Foreign tradable goods, $i = 1...N$ and $i^* = 1...N^*$:

$$p_{NT} Y_{NT}^d = (1 - \beta)I; \quad p_{NT^*} Y_{NT^*}^d = (1 - \beta)I^* \quad (17)$$



$$Y_i^d = a^\sigma \left(\frac{p_i}{P_T} \right)^{-\sigma} \beta \frac{I + I^*}{P_T}; \quad Y_{i^*}^d = a^*{}^\sigma \left(\frac{p_{i^*}}{P_T} \right)^{-\sigma} \beta \frac{I + I^*}{P_T} \quad (18)$$

where Y_j^d and $Y_{j^*}^d$ are the total demand for goods $j = i, NT$ and $j^* = i^*, NT^*$, I and I^* the total income in the Home and Foreign country, and P_T the price of tradable goods:

$$P_T = \left(a^\sigma \sum_{j=1}^N p_j^{1-\sigma} + a^*{}^\sigma \sum_{j^*=1}^{N^*} p_{j^*}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (19)$$

Equations (18) determine the price of the Home tradable good i relative to the price of the Foreign tradable good i^* as:

$$\frac{p_i}{p_{i^*}} = \frac{a}{a^*} \left(\frac{Y_{i^*}^d}{Y_i^d} \right)^{1/\sigma} \quad (20)$$

Production and factor demand

Non-tradable service

The markets for the non-tradable services being perfectly competitive, the zero profit condition entails $p_{NT} = w_L$ and $p_{NT^*} = w_L^*$. Inserting these equations and the production functions $Y_{NT} = L_{NT}$ and $Y_{NT^*}^* = L_{NT^*}^*$ into Eqs. (17) yields $w_L L_{NT} = (1 - \beta)I$ and $w_L^* L_{NT^*}^* = (1 - \beta)I^*$. As $I = w_L \bar{L} + w_H \bar{H}$ and $I^* = w_L^* \bar{L}^* + w_H^* \bar{H}^*$, one obtains the following demand for unskilled labour in the non-tradable sector in each country:

$$L_{NT} = (1 - \beta) \left(\bar{L} + w \bar{H} \right); \quad L_{NT^*}^* = (1 - \beta) \left(\bar{L}^* + w^* \bar{H}^* \right) \quad (21)$$

with $w \equiv w_H/w_L$ and $w^* \equiv w_H^*/w_L^*$ being the skill premia in the Home and Foreign country.

Tradable goods

The tradable goods i and i^* are, respectively, produced in Home and Foreign country with the Cobb–Douglas technologies:

$$Y_i = A L_i^\alpha H_i^{1-\alpha}, \quad i = 1 \dots N; \quad Y_{i^*} = A^* (L_{i^*}^*)^\alpha (H_{i^*}^*)^{1-\alpha}, \quad i^* = 1 \dots N^* \quad (22)$$

At the optimum, the skill premia are $w = \frac{1-\alpha}{\alpha} \frac{L_i}{H_i}$, $i = 1 \dots N$ and $w^* = \frac{1-\alpha}{\alpha} \frac{L_{i^*}^*}{H_{i^*}^*}$, $i^* = 1 \dots N^*$. Consequently, $\frac{L_i}{H_i} = \frac{L_{j^*}}{H_{j^*}} = \frac{L_T}{H_T}$, $\forall i, j \in [1 \dots N]$ and $\frac{L_{i^*}^*}{H_{i^*}^*} = \frac{L_{j^*}^*}{H_{j^*}^*} = \frac{L_T^*}{H_T^*}$, $\forall i^*, j^* \in [1 \dots N^*]$, where L_T and H_T (resp. L_T^* and H_T^*) are the total

amounts of unskilled and skilled labour utilised in the Home (resp. Foreign) tradable sector. Finally:

$$L_T = \frac{\alpha}{1-\alpha} w H_T \Leftrightarrow w = \frac{1-\alpha}{\alpha} \frac{L_T}{H_T} \quad (23)$$

$$L_T^* = \frac{\alpha}{1-\alpha} w^* H_T^* \Leftrightarrow w^* = \frac{1-\alpha}{\alpha} \frac{L_T^*}{H_T^*}$$

Equilibrium

Perfect competition in the four labour markets (L, H, L^*, H^*) entails $\bar{L} = L_{NT} + L_T$, $\bar{H} = H_T$, $\bar{L}^* = L_{NT}^* + L_T^*$ and $\bar{H}^* = H_T^*$.

Combining (23), the four factor equilibrium equations and (21), we obtain:

$$w = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} \frac{\bar{L}}{\bar{H}}; w^* = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} \frac{\bar{L}^*}{\bar{H}^*} \quad (24)$$

$$L_T = \frac{\alpha\beta}{1-(1-\alpha)\beta} \bar{L}; L_T^* = \frac{\alpha\beta}{1-(1-\alpha)\beta} \bar{L}^* \quad (25)$$

$$L_{NT} = \frac{1-\beta}{1-(1-\alpha)\beta} \bar{L}; L_{NT}^* = \frac{1-\beta}{1-(1-\alpha)\beta} \bar{L}^* \quad (26)$$

The demand functions (18) and the production functions (22) being identical for all tradable goods in each country, we have $Y_i = Y_j$ and hence $L_i = L_j = L_T/N$ and $H_i = H_j = \bar{H}/N$, $\forall i, j \in [1...N]$, in the Home country, and $Y_{i^*} = Y_{j^*}$, $L_{i^*} = L_{j^*} = L_T^*/N^*$ and $H_{i^*} = H_{j^*} = \bar{H}^*/N^*$, $\forall i^*, j^* \in [1...N^*]$ in the Foreign country. Hence, because of Eqs. (22) to (A8) and the assumption of given relative factor endowments $\bar{\lambda} = \bar{H}/\bar{L}$ and $\bar{\lambda}^* = \bar{H}^*/\bar{L}^*$:

$$Y_i = \frac{A}{N} \left(\frac{\alpha\beta}{1-(1-\alpha)\beta} \right)^\alpha \bar{\lambda}^{1-\alpha} \bar{L}, i = 1...N \quad (27)$$

$$Y_{i^*} = \frac{A^*}{N^*} \left(\frac{\alpha\beta}{1-(1-\alpha)\beta} \right)^\alpha \bar{\lambda}^{1-\alpha} \bar{L}^*, i^* = 1...N^*$$

Combining Eqs. (20), (27) and the equilibria $Y_i = Y_i^d$ and $Y_{i^*} = Y_{i^*}^d$ yield:

$$\frac{p_T}{p_T^*} = \frac{a}{a^*} \left(\frac{N}{N^*} \right)^{1/\sigma} \left(\frac{A^*}{A} \right)^{1/\sigma} \left(\frac{\bar{\lambda}^*}{\bar{\lambda}} \right)^{\frac{1-\alpha}{\sigma}} \left(\frac{\bar{L}^*}{\bar{L}} \right)^{1/\sigma} \quad (28)$$

At the firms' optimum with a Cobb–Douglas technology, we have :



$$p_T = \frac{w^{1-\alpha}}{A\alpha^\alpha(1-\alpha)^{1-\alpha}} w_L \Leftrightarrow w_L = \frac{A\alpha^\alpha(1-\alpha)^{1-\alpha}}{w^{1-\alpha}} p_T$$

$$p_T^* = \frac{w^*{}^{1-\alpha}}{A^* \alpha^\alpha(1-\alpha)^{1-\alpha}} w_L^* \Leftrightarrow w_L^* = \frac{A^* \alpha^\alpha(1-\alpha)^{1-\alpha}}{w^*{}^{1-\alpha}} p_T^*$$

And because of Eq. (24):

$$w_L = A \frac{\alpha^\alpha(1-(1-\alpha)\beta)^{1-\alpha}}{\beta^{1-\alpha}} \bar{\lambda}^{1-\alpha} p_T; w_L^* = A^* \frac{\alpha^\alpha(1-(1-\alpha)\beta)^{1-\alpha}}{\beta^{1-\alpha}} \bar{\lambda}^*{}^{1-\alpha} p_T^* \quad (29)$$

Combining Eqs. (28) and (29):

$$\frac{w_L}{w_L^*} = \frac{a}{a^*} \left(\frac{N}{N^*} \right)^{1/\sigma} \left(\frac{A}{A^*} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\bar{\lambda}}{\bar{\lambda}^*} \right)^{(1-\alpha)\frac{\sigma-1}{\sigma}} \left(\frac{\bar{L}}{\bar{L}^*} \right)^{1/\sigma} \quad (30)$$

By adding the labour force dynamics $\bar{L}_t = e^{n \times t} \bar{L}_0$ and $\bar{L}_t^* = e^{n^* \times t} \bar{L}_0^*$ to Eqs. (29) and (30), and denoting $\bar{\gamma} = \frac{n^*-n}{\sigma} > 0$, we determine Eqs. (7) and (8) in the text. Equation (9) is determined by combining Eqs. (30), (24) and the labour force dynamics.

Equation (5) determines the countries' consumer price: $P = (p_{NT})^{1-\beta} (p_T)^\beta$; $P^* = (p_{NT}^*)^{1-\beta} (p_T^*)^\beta$.

As $p_{NT} = w_L$ and $p_{NT}^* = w_L^*$, we have: $P = (p_T)^\beta (w_L)^{1-\beta}$, $P^* = (p_T^*)^\beta (w_L^*)^{1-\beta}$ and finally: $P/P^* = (w_L/w_L^*)^{1-\beta}$.

As $\frac{w_L}{w_L^*}$ grows at rate $\bar{\gamma} = \frac{n^*-n}{\sigma}$, P/P^* grows at rate $(1-\beta)\bar{\gamma}$.

The relative real wages are $\frac{v_L}{v_L^*} = \frac{w_L/P}{w_L^*/P^*} = \left(\frac{w_L}{w_L^*} \right)^\beta$ for unskilled labour and $\frac{v_H}{v_H^*} = \frac{w_H/P}{w_H^*/P^*} = \frac{w \times w_L/P}{w^* \times w_L^*/P^*} = \frac{w}{w^*} \left(\frac{w_L}{w_L^*} \right)^\beta$ for skilled labour. As $\frac{w_L}{w_L^*}$ grows at rate $\bar{\gamma}$, $\frac{v_L}{v_L^*}$ grows at rate $\beta\bar{\gamma}$, as well as $\frac{v_H}{v_H^*}$ since $\frac{w}{w^*}$ is constant.

Finally, the relative real income per head i/i^* is equal to $\frac{(v_L \bar{L} + v_H \bar{H})/(\bar{L} + \bar{H})}{(v_L^* \bar{L}^* + v_H^* \bar{H}^*)/(\bar{L}^* + \bar{H}^*)} = \frac{(\bar{L} + w \bar{H})/(\bar{L} + \bar{H})}{(\bar{L}^* + w^* \bar{H}^*)/(\bar{L}^* + \bar{H}^*)} \times \frac{v_L}{v_L^*} = \frac{\bar{L}/(\bar{L} + \bar{H})}{\bar{L}^*/(\bar{L}^* + \bar{H}^*)} \times \frac{v_L}{v_L^*}$. As $\frac{\bar{L}}{\bar{L} + \bar{H}}$ and $\frac{\bar{L}^*}{\bar{L}^* + \bar{H}^*}$ are constant, i/i^* grows at the same rate $\beta\bar{\gamma}$ as v_L/v_L^* .



Appendix B: Proof of Proposition 4

We denote $\varpi(t) \equiv w_L(t)/w_L^*(t)$ the exogenous relation which binds the Foreign unskilled wage to that of the Home country, $\varpi(t)$ being lower than the full employment value determined by Eq. (8), and $\tilde{\gamma} < \bar{\gamma} = (n^* - n)/\sigma$ denoting its exogenous growth rate.

The above-defined relations (17) – (30) are still valid except that $\varpi(t) \equiv w_L(t)/w_L^*(t)$ is now exogenous and \bar{L}^* and $\bar{\lambda}^*$ must be replaced by $L^*(t)$ and $\lambda^*(t) \equiv \bar{H}^*(t)/L^*(t)$, where $L^*(t) = (L_{NT}^*(t) + L_T^*(t)) < \bar{L}^*(t)$ is the total Foreign unskilled employment at time t .

Proof of feature 1

The modified Eq. (30) is:

$$\varpi(t) = \frac{a}{a^*} \left(\frac{N}{N^*} \right)^{\frac{1}{\sigma}} \left(\frac{A}{A^*} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{\bar{\lambda}}{\bar{H}^*(t)} \right)^{(1-\alpha)\frac{\sigma-1}{\sigma}} (L^*(t))^{1-\alpha\frac{\sigma-1}{\sigma}} (\bar{L}(t))^{-\frac{1}{\sigma}} \quad (31)$$

Hence:

$$L^*(t) = \bar{C} \left((\varpi(t))^\sigma \times (\bar{H}^*(t))^{(1-\alpha)(\sigma-1)} \times \bar{L}(t) \right)^{\frac{1}{(1-\alpha)(\sigma-1)+1}} \quad (32)$$

where $\bar{C} \equiv \left(\left(\frac{a^*}{a} \right)^\sigma \frac{N}{N^*} \left(\frac{A^*}{A} \right)^{\sigma-1} \frac{1}{\bar{\lambda}} \right)^{\frac{1}{(1-\alpha)(\sigma-1)+1}}$ is a constant term.

Since $\varpi(t) = \varpi(0) \times e^{\tilde{\gamma}t}$, $\bar{H}^*(t) = \bar{H}^*(0) \times e^{n^*t}$ and $\bar{L}(t) = \bar{L}(0) \times e^{nt}$, then:

$$L^*(t) = \bar{C} \left((\varpi(0))^\sigma (\bar{H}^*(0))^{(1-\alpha)(\sigma-1)} \bar{L}(0) \right)^{\frac{1}{(1-\alpha)(\sigma-1)+1}} \times e^{\frac{\sigma\tilde{\gamma} + (1-\alpha)(\sigma-1)n^* + n}{(1-\alpha)(\sigma-1)+1}t} \quad (33)$$

Denote the growth rate of L^* as:

$$\tilde{n}_L^* = \frac{\sigma\tilde{\gamma} + (1-\alpha)(\sigma-1)n^* + n}{(1-\alpha)(\sigma-1)+1} \quad (34)$$

By assumption, $\tilde{\gamma} < \bar{\gamma} = (n^* - n)/\sigma$. Inserting $\bar{\gamma}$ as an upper bound in Eq. (34) leads to $\frac{\sigma\bar{\gamma} + (1-\alpha)(\sigma-1)n^* + n}{(1-\alpha)(\sigma-1)+1} = n^*$. Hence, $\tilde{n}_L^* < n^*$.

Proof of feature 2

Because of Eqs. (24), $w = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} \frac{1}{\bar{\lambda}}$ and $w^*(t) = \frac{(1-\alpha)\beta}{1-(1-\alpha)\beta} \frac{1}{\lambda^*(t)}$, with $\lambda^*(t) \equiv \bar{H}^*(t)/L^*(t)$. An increase in ϖ which is lower than its full employment value entails an increase in λ^* and thereby a decrease in the skill premium w^* and an increase in the relative skill premia w/w^* since full employment in the Home country keeps w constant over time.



Appendix C: Proof of Proposition 5

Proof of feature 1

The transfer increases the Foreign income and decreases the Home income by the same amount φ . Let Y_i^d and Y_i^{d*} denote the demands for tradable good i by the Home and Foreign country, and Y_{i*}^d and Y_{i*}^{d*} the demands for tradable good i^* by the Home and Foreign country. Y_i and Y_{i*} denote the supply of goods i and i^* , and Y_{NT*} the supply of the foreign service NT^* . The equality ‘income = expenditure’ in the Foreign country entails $\varphi + p_{NT*} Y_{NT*} + \sum_{i*=1}^{N*} p_{i*} Y_{i*} = \sum_{i*=1}^{N*} p_{i*} Y_{i*}^{d*} + \sum_{i=1}^N p_i Y_i^{d*} + p_{NT*} Y_{NT*}^{d*}$, and as the market for NT^* is balanced ($Y_{NT*}^* = Y_{NT*}^{d*}$), we have $\varphi + \sum_{i*=1}^{N*} p_{i*} Y_{i*} = \sum_{i*=1}^{N*} p_{i*} Y_{i*}^{d*} + \sum_{i=1}^N p_i Y_i^{d*}$. The markets for goods $i^* = 1 \dots N^*$ are balanced: $\sum_{i*=1}^{N*} p_{i*} Y_{i*} = \sum_{i*=1}^{N*} p_{i*} Y_{i*}^{d*} + \sum_{i=1}^N p_i Y_i^{d*}$. Combining both equalities yields: $\varphi = \sum_{i=1}^N p_i Y_i^{d*} - \sum_{i*=1}^{N*} p_{i*} Y_{i*}^d$, where $\sum_{i=1}^N p_i Y_i^{d*}$ are the Foreign imports and $\sum_{i*=1}^{N*} p_{i*} Y_{i*}^d$ the Foreign exports. This establishes feature 1.

Proof of feature 2

see Online Appendix 1.

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Declarations

Conflict of interest The authors declares that there is no conflict of interest to disclose.

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